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$\bar{B}-B$ Mixing in the Large N_C Expansion

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Abstract

The large N_C expansion is used to analyze the long and short distance contributions to the B-parameter describing the weak mixing of B-mesons. An approximate matching condition may allow a consistent estimate of the full B-parameter. Application of similar methods to the B-parameter and weak decay amplitudes in the K-meson system and the pion electromagnetic mass difference are reviewed.

Introduction.

Weak mixing amplitudes involve hadronic matrix elements of the effective weak Hamiltonian. At low energies, the effective Hamiltonian may be written as a product of weak currents. The structure of the hadronic matrix elements of these operators greatly simplifies using a large N_C expansion. This expansion exists in both the quark-gluon description of QCD and in the effective meson theory. At leading order, the amplitudes factorize and at next-to-leading order the amplitudes have a simple structure which can be exploited to match a short distance description in terms of quarks and gluons to a long distance description in terms of an effective chiral Lagrangian for the heavy and light mesons.

We begin by discussing the weak mixing structure of the weak mixing amplitudes for B-mesons. The application of the large N_C expansion to these processes is discussed. The leading nonfactorizing contributions to the amplitudes are analyzed in terms of a dual description involving a quark-gluon picture and a meson picture. The duality and approximate matching of these pictures are discussed along with a preliminary estimate for the B-meson B–parameter.

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Similar methods have been previously applied in other situations where the effective Hamiltonian has a current-current structure. We review the computation of the K-meson B-parameter and the weak decay amplitudes. The application of these methods to the pion electromagnetic mass difference illustrates the power of the method and the directions where the explicit computations could be improved.

Weak mixing amplitudes for B-mesons.

At low energy, the effective weak Hamiltonian describing the weak mixing is generated by the electroweak loop amplitudes. Integrating out the heavy particles - W-bosons, top quarks, etc. - and using perturbative QCD at short distance, weak mixing processes are described by an effective Hamiltonian,

$$H_{eff} = \sum_i C_i(\mu) \cdot Q_i(\mu) \tag{1}$$

where $C_i(\mu)$ is a coefficient function depending on the weak mixing angles, the heavy particle masses, the weak coupling constants and a QCD normalization scale, μ. $Q_i(\mu)$ are four-fermion operators involving the effective light degrees of freedom.

The operator structure of the effective Hamiltonian for the weak mixing of B-mesons involves the product of color singlet weak currents,

$$\begin{aligned} Q_d &= \bar{b}\gamma^\mu(1-\gamma_5)d \cdot \bar{b}\gamma_\mu(1-\gamma_5)d \\ Q_s &= \bar{b}\gamma^\mu(1-\gamma_5)s \cdot \bar{b}\gamma_\mu(1-\gamma_5)s \end{aligned} \tag{2}$$

The B-parameters are defined through the weak matrix elements,

$$\begin{aligned} \tfrac{8}{3} f_B^2 m_B^2 \cdot B_{B_d} &\equiv \left\langle \bar{B}^o(v) \middle| \bar{b}\gamma^\mu(1-\gamma_5)d \cdot \bar{b}\gamma_\mu(1-\gamma_5)d \middle| B^o(v) \right\rangle \\ \tfrac{8}{3} f_B^2 m_B^2 \cdot B_{B_s} &\equiv \left\langle \bar{B}_s(v) \middle| \bar{b}\gamma^\mu(1-\gamma_5)s \cdot \bar{b}\gamma_\mu(1-\gamma_5)s \middle| B_s(v) \right\rangle \end{aligned} \tag{3}$$

In this definition, the B-parameter depends on the scale, μ, used to define the effective Hamiltonian and contains all nonperturbative effects associated with the formation of the hadronic states. Since the B-meson is a bound state of the heavy b-quark and light quarks and gluons, the scale, μ, may be chosen below the b-quark mass and above the QCD scale. All nonperturbative effects should be associated with the dynamics of the light quarks and gluons; the b-quark acts as a static color source except for a local interaction which changes the b-quark to an anti-b-quark.

The large N_C expansion may be used to analyze the structure of the weak matrix elements. The large N_C expansion is a formal expansion where the strong coupling constant tends to zero as the number of colors, N_C, with the effective planar coupling constant, $\alpha_p \equiv \alpha_s \cdot N_c$, held fixed. At leading order, the theory consists of the sum over all color singlet planar quark-gluon diagrams. Mesons are color singlet, nonperturbative bound states of the quarks and gluons. In this limit, the planar diagrams describe a weakly interacting theory of meson bound states formulated at tree level. Higher order diagrams describe the meson loop amplitudes and are suppressed by powers of $1/N_C$.

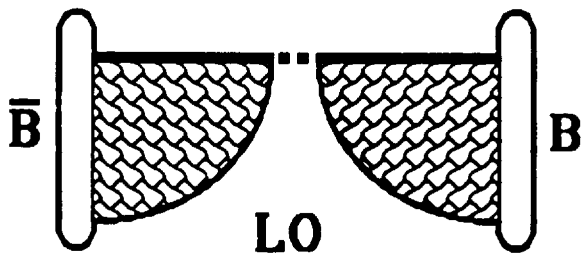

Figure 1. Diagrams contributing to the leading order in the large N_C expansion where the hatched area represents planar insertions of arbitrary numbers of gluons.

At leading order, the weak mixing amplitudes factorize as shown in Figure 1. The currents independently produce the meson states, and the strong dynamics does not generate correlations. The nonperturbative physics is contained in the individual current matrix elements and is expressed in the values of the weak decay constants, $\bar{f}_B$. The matrix element for the B_d meson is given by

$$\begin{aligned}
&\left\langle \bar{B}^o(v) \middle| \bar{b}\gamma^\mu(1-\gamma_5)d \cdot \bar{b}\gamma_\mu(1-\gamma_5)d \middle| B^o(v) \right\rangle \\
&= 2\left\langle \bar{B}^o(v) \middle| \bar{b}\gamma^\mu(1-\gamma_5)d \middle| 0 \right\rangle \cdot \left\langle 0 \middle| \bar{b}\gamma_\mu(1-\gamma_5)d \middle| B^o(v) \right\rangle \\
&= 2\bar{f}_B^2 m_B^2 \quad \Rightarrow \quad B_{B_d^o} = \frac{3}{4} \quad (LO)
\end{aligned} \tag{4}$$

The B-parameter at leading order is 3/4 since the usual definition of the factorized amplitude includes a contribution from the Fierz terms which are properly included in the $1/N_C$ corrections. Similar results are obtained for the B_S meson B- parameter.

Nonfactorizing Contributions to the B-parameter.

At the next-to-leading order, NLO, the $1/N_C$ corrections generate nonfactorizing contributions to the weak matrix elements. However, these amplitudes can be written as integrals over the connected current correlation function,

$$A = \int dq \langle \overline{B}^o(v) | J^\mu(q) \cdot J_\mu(-q) | B^o(v) \rangle_{connected} \tag{5}$$

where $J_\mu(q) = \{\bar{b}\gamma_\mu(1-\gamma_5)d\}(q)$. The structure of the nonfactorizing amplitudes are shown in Figure 2. Again, the hatched areas correspond to planar insertions of arbitrary numbers of gluons with a twist shown at the bottom of the figure. Before integrating over the momentum, q, flowing through currents, the amplitude has a tree level structure in the meson picture of the large N_C expansion. The integration produces meson loop amplitudes.

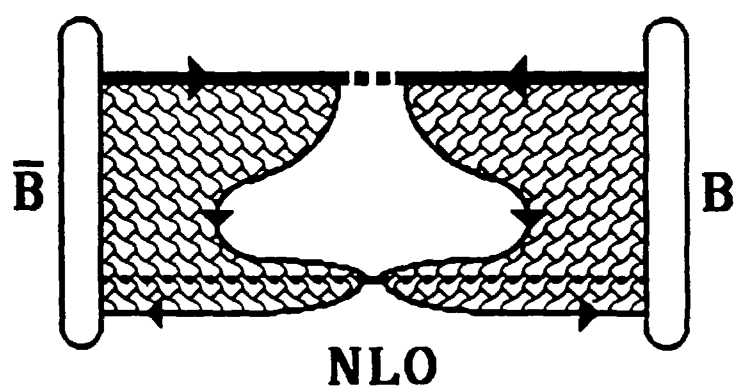

Figure 2. Diagrams contributing to the next-to-leading order in the large N_C expansion where the hatched area represents planar insertions of arbitrary numbers of gluons.

At the quark level, the b-quark carries a large four-momentum associated with its heavy mass, $p_\mu = m_b \cdot v_\mu$. The momentum integration of Equation (5) may be separated into several regions where different pictures may be used to estimate the value amplitude in each region. Defining $q_\mu = m_b v_\mu + k_\mu$, the integration regions are,

- $k > m_b$ - use short distance picture of normal perturbative QCD
- $\Lambda_{QCD} < k < m_b$ - use heavy quark effective theory of perturbative QCD
- $k < \Lambda_{QCD}$ - use long distance picture of the effective meson theory.

In our calculations, we will take the scale, μ, in the range below the b-quark mass so that the perturbative heavy quark effective theory describes the explicit short distance physics with other short distance effects absorbed in the coefficient functions of the effective Hamiltonian of Equation 1.

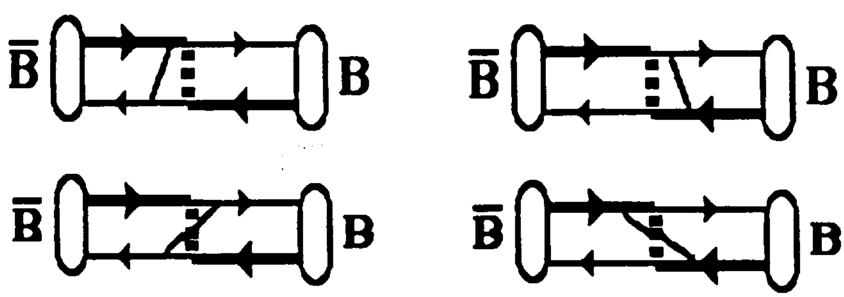

Figure 3. Leading short distance contributions to the nonfactorizing weak mixing amplitudes for B_d mesons. The diagonal lines represent gluon exchange corrections to the weak vertex.

Using the perturbative heavy quark effective theory, the nonfactorizing contributions to the weak mixing amplitudes come from the diagrams shown in Figure 3. The leading corrections are the one gluon exchange corrections to the current-current matrix elements. The NLO short distance amplitude is given by perturbative QCD,

$$A = A_{LO} \cdot \frac{4\pi\alpha_s}{N_c} C_F \frac{i}{(2\pi)^4} \int dk \left[\frac{2}{(k^2)^2} + \frac{1}{k^2 (v \cdot k)^2} \right] \qquad (6)$$

where $C_F = (N_c^2 - 1)/2N_c$. This amplitude has logarithmic infrared divergences which must be matched to a similar behavior of the long distance contributions to the integrand.

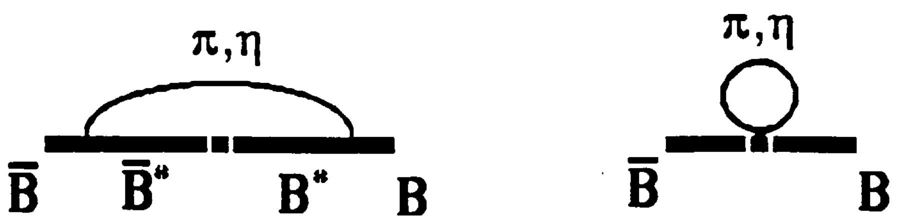

Figure 4. Leading long distance contributions to the nonfactorizing weak mixing amplitudes for B mesons. π and η mesons contribute in the loops.

The leading long distance contributions to the nonfactorizing amplitudes involve pseudoscalar meson loops with B and B* mesons poles in the current amplitudes as shown in Figure 4. To describe the long distance contributions, a chiral Lagrangian is used to describe the interaction of heavy mesons, B and B*, with the pseudoscalar mesons [1]. At lowest order in a momentum expansion, this Lagrangian has the form,

$$\begin{aligned} L = f_\pi^2\Big[\tfrac{1}{4}tr\{\partial^\mu\Sigma\partial_\mu\Sigma\} + \tfrac{1}{4}r_o\cdot tr\{m_q(\Sigma+\Sigma^\dagger)\}\Big] \\ - itr\{\ \overline{H}^a v^\mu\partial_\mu H_a - \tfrac{1}{2}\cdot\overline{H}^a v^\mu H_b\big[\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\big] \\ -\tfrac{1}{2}g\overline{H}^a H_b\gamma^\mu\gamma_5\big[\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\big]\ \} + \cdots \end{aligned} \tag{7}$$

where $H_a = \dfrac{1+\gamma\cdot v}{2}\cdot\left[B^*_{a\mu}\gamma^\mu - B_a\gamma_5\right]$, $\quad \Sigma = \exp\{i\lambda\cdot\Pi / f_\pi\}$.

In the chiral limit, the NLO contribution is given by,

$$A = A_{LO}\frac{i}{(2\pi)^4}\int dk\ \{-\frac{2}{3f_\pi^2}(1-g^2)\frac{1}{k^2} - \frac{2}{3f_\pi^2}g^2\frac{1}{(v\cdot k)^2}\}. \tag{8}$$

This amplitude is infrared finite but is divergent in the ultraviolet. It represents the exact low momentum behavior of the integrand of Equation 5. At higher momenta, this amplitude must match the behavior of Equation 6 which gives the correct short distance behavior in the region where the heavy quark effective theory applies.

The quark-gluon picture of QCD and the hadronic (meson) theory of QCD are dual descriptions of the same physics. Perturbation theory may be used in the quark-gluon picture to describe the short distance behavior of the theory. The loop expansion of the chiral Lagrangian can be used to describe the long distance behavior. As the momentum scale is varied from short to long distance, the current-current amplitude used to describe the weak mixing processes at $O(1/N_C)$ must be a continuous function of scale. A consistent merging of the two pictures requires a matching at intermediate scales. In our case, the matching must reproduce the proper analytic structure of the functions as functions of the two variables, k^2 and $v \cdot k$. If we use the minimal approximations given in Equation 6 and Equation 8, then a relatively sharp cutoff is required as it is known that both approximations rapidly breakdown outside their domain of validity. Indeed, there may be no region of overlap.

To achieve an approximate matching, we use an example of an analytic matching procedure. We introduce an arbitrary separation of long and short distance effects by introducing the following resolution of 1 into Equation 5,

$$1 = \frac{k^2(k^2 - 2\lambda^2)}{(k^2 - \lambda^2)^2}\ (SD) + \frac{\lambda^4}{(k^2 - \lambda^2)^2}\ (LD) \tag{9}$$

The two terms in Equation 9 define regularized amplitudes. The short distance contribution takes the first term (SD) and inserts it into the integral of Equation 6. The infrared divergences are regularized and a finite value of the integral is obtained. The long distance contribution takes the second term (LD) and inserts it into the integral of Equation 8. The ultraviolet divergences are regularized in this case, and a finite value of the integral is also achieved. The two contributions are then added together to evaluate the full value of Equation 5. The regularization methods merely allow appropriate approximations to be made in each case.

If we separate the k^2 and the $v \cdot k$ contributions, we find, respectively,

$$A_{reg} = -\frac{2}{3f_\pi^2}(1-g^2)\cdot\frac{\lambda^2}{16\pi^2} - 2\pi\alpha_s\frac{2}{16\pi^2}\ln\left(\frac{\mu^2}{\lambda^2}\right) + const \tag{10}$$

$$A_{reg} = -\frac{2}{3f_\pi^2}(-2g^2)\cdot\frac{\lambda^2}{16\pi^2} + 2\pi\alpha_s\frac{2}{16\pi^2}\ln\left(\frac{\mu^2}{\lambda^2}\right) + const \tag{11}$$

A consistent matching of these two amplitudes can be achieved if

$$\lambda^2 = 6\pi\alpha_s\frac{f_\pi^2}{(1-g^2)} = 6\pi\alpha_s\frac{f_\pi^2}{(2g^2)} \quad \Rightarrow \quad g^2 = \frac{1}{3} \tag{12}$$

and $\lambda \approx \sqrt{8\pi} f_\pi \cdot \sqrt{\alpha_s} \approx 660 MeV \cdot \sqrt{\alpha_s}$ which is similar to the matching condition found for the K-meson decay amplitudes discussed below. While it is possible to achieve consistent results, there could still be large corrections if the low-lying states do not saturate the matching conditions. With this caution, we proceed and combine the two terms in Eq. 10 and Eq. 11 to give the full nonfactorizing contributions to the B-parameter. We note that the short distance contributions cancel between the two terms which is consistent with the vanishing of the nonfactorizing anomalous dimension of the effective weak Hamiltonian in the heavy quark effective theory [2].

To compute the B-parameter, we must keep the full infrared dependence on the appropriate meson masses. We use the static limit for the heavy quark in this analysis. For B^0 mesons, η and π mesons contribute in the loops with the scale, $\mu < m_B$, which predicts the nonfactorizing corrections to the weak mixing amplitude

$$\begin{aligned} A(B^o) &= \left\langle \overline{B}^o(v) \middle| (\bar{b}\gamma^\mu(1-\gamma_5)d \cdot (\bar{b}\gamma^\mu(1-\gamma_5)d \middle| B^o(v) \right\rangle \\ &= 2\bar{f}_B^2 m_B^2 \left[1 - \frac{1}{6f_\pi^2} \cdot (1-3g^2) \cdot [J_2(m_\eta,\lambda) + 3 \cdot J_2(m_\pi,\lambda)] \right] \end{aligned} \tag{13}$$

where

$$J_2(m,\lambda) = \frac{1}{16\pi^2} \cdot \frac{\lambda^2}{\lambda^2 - m^2} \cdot \left[\lambda^2 - m^2 \frac{\lambda^2}{\lambda^2 - m^2} \ln(\lambda^2 / m^2) \right] \tag{14}$$

For B_S mesons, only the η meson contributes in the loops for $\mu < m_B$ which predicts the amplitude

$$\begin{aligned} A(B_s) &= \left\langle \overline{B}_s(v) \middle| (\bar{b}\gamma^\mu(1-\gamma_5)s) \cdot (\bar{b}\gamma_\mu(1-\gamma_5)s) \middle| B_s(v) \right\rangle \\ &= 2\bar{f}_B^2 m_B^2 \left[1 - \frac{4}{6f_\pi^2} (1-3g^2) \cdot J_2(m_\eta,\lambda) \right] \end{aligned} \tag{15}$$

The numerical results are

$$B_{B_d} = \tfrac{3}{4}\left[1 - 0.10 \cdot (1-3g^2)\right] \;\rightarrow\; \frac{3}{4} \tag{16}$$

$$B_{B_s} = \tfrac{3}{4}\left[1 - 0.07 \cdot (1 - 3g^2)\right] \;\to\; \frac{3}{4} \tag{17}$$

with $\lambda = 700 MeV$, $f_\pi = 131 MeV$. Note that the consistent matching conditions of Eq. 12 require $\left(1 - 3g^2\right) \to 0$.

This result agrees with the chiral log calculation of these amplitudes [3] where the short distance consistency was not attempted. The result appears to differ from the result of a sum rule calculation where the fully factorized result, B = 1, is obtained with small corrections [4]. In this calculation, the condensate contributions are estimated for the corrections to (B-1) and are shown to be small. The B-parameter has also been estimated using lattice field theory methods where it is shown that B_{LL} = 1 to within a few percent [5]. Earlier results had been obtained for the B-meson weak mixing parameters using harmonic oscillator and Bag models for the heavy-light quark bound states [6]. The effective B-parameter was found to be in the range, B = R_B = 0.5-0.8.

The consistency of our approximation demands that $\left(1 - 3g^2\right) \to 0$ so that the NLO corrections to the B-parameter vanish, and the LO approximation to the large N_c expansion is retained. If further states are included in the low energy approximation, it may be possible to relax this condition on the pseudovector coupling constant. In particular, the B^{**} (1/2+) mesons are strongly coupled to the $B - \pi$ system and may be important to include in a complete analysis [7]. The duality properties of these amplitudes are more complex than the usual current correlation functions as the amplitudes depend on two invariants, k^2 and $v \cdot k$. A better understanding of local duality is essential for achieving a precise matching condition between the hadronic calculations and those of the heavy quark effective field theory [8]. We hope to clarify these duality properties in the future. However, it is remarkable that even the minimal approximations to the long and short distance structure of the nonfactorizing contributions lead to a coherent picture of the weak mixing amplitudes in the $1/N_c$ expansion.

Applications to K-meson amplitudes.

The large N_c expansion combined with the matching of long and short distance structure has previously been applied in the K-meson system [9]. The long distance behavior is described by an $SU(3) \otimes SU(3)$ chiral Lagrangian while the short distance behavior is described by normal perturbative QCD. The calculation is a simplified version of that described above for the B-meson system since the s-quark can be treated as a massive but light quark.

The B-parameter for $K^0 - \bar{K}^0$ mixing is computed using a consistent matching scale of 600-800 MeV. The invariant B-parameter is given by

$$\hat{B}_K = 0.66 \pm 0.10 \tag{18}$$

where the "error" includes an estimate of the systematic errors of the method [10]. The matching conditions can be improved by including vector mesons in the effective long distance approximation to the mixing amplitudes. This makes the extrapolation of the long distance theory more accurate and extends the range of matching to the short distance behavior of the quark-gluon picture. This improvement tends to increase slightly the value of the B-parameter with the result

$$\hat{B}_K = 0.70 \pm 0.10 \tag{19}$$

The K-meson B-parameter has also been computed by various lattice groups. Using a quadratic extrapolation of the lattice data, the STAG Collaboration [11] obtains

$$\begin{aligned} &B_K(NDR, 2GeV) = 0.616 \pm .020 \pm .017 \\ &\Rightarrow \quad \hat{B}_K = 0.825 \pm 0.027 \pm .023 \end{aligned} \tag{20}$$

The JLQCD Collaboration [12] presented preliminary results at the Lattice '95 Conference which argued for the necessity of a linear extrapolation to the continuum limit in their data and reported a new value for the B-parameter,

$$\begin{aligned} &B_K(NDR, 2GeV) = 0.497 \pm .008 \pm .014 \\ &\Rightarrow \quad \hat{B}_K = 0.67 \pm \ \ldots \end{aligned} \tag{21}$$

The results of the large N_C analysis of the B-parameter are clearly consistent with the trend of these lattice computations.

The large N_C analysis has also been applied to the nonleptonic weak decay amplitudes for K-mesons, eg. $K \to n\pi$. A consistent matching is found connecting the chiral Lagrangian calculation with the short distance behavior predicted by perturbative QCD. The $\Delta I = 3/2$ amplitudes are found to be suppressed and are in agreement with the experimental values of the weak decay amplitudes. A significant enhancement of the $\Delta I = 1/2$ amplitudes is obtained from a combination of the short distance evolution, the long distance matrix elements and penguin effects [13]. Detailed agreement with the current experimental data for these amplitudes will require further analysis of the various elements of the calculation and an improvement of the matching conditions.

Application to the pion electromagnetic mass difference.

The $1/N_C$ expansion can also be applied to the virtual photon corrections to the pion electromagnetic mass difference which requires the pionic matrix elements of a current-current amplitude integrated with the photon propagator. The long-distance contribution can be computed using the pion chiral Lagrangian and matched to the short distance contributions computed using perturbative QCD and nonperturbative condensates [14]. These contributions are found to be

$$LD: \quad \Delta m_\pi^2 = \frac{3}{4\pi}\alpha_{em}\int dq^2 \tag{22}$$

$$SD: \quad \Delta m_\pi^2 = 6\frac{\alpha_{em}}{f_\pi^2}\cdot\int\frac{dq^2}{q^4}\ \alpha_s\left(\langle 0|\bar{q}q|0\rangle\right)^2 \tag{23}$$

A crude matching of the long and short distance contributions gives an estimate of about 5-6 MeV for the $\pi^+ - \pi^o$ mass difference, depending on the values of Λ_{QCD} and the value of the chiral condensate. The amount of the experimental mass difference attributed to the photon exchange process is

$$\Delta m_\pi = 4.43 \pm 0.03\ \ MeV \tag{24}$$

which is in reasonable agreement with the above estimate.

The above calculation may be improved by including vector mesons and axial-vector mesons in the description of the long distance amplitudes. The evolution is now found to exactly match the short distance structure at scales above 1 GeV. Replacing Eq. 22, the long distance contribution becomes

$$LD: \quad \Delta m_\pi^2 = \frac{3}{4\pi}\alpha_{em}\int dq^2\,\frac{m_A^2 m_V^2}{(q^2+m_A^2)(q^2+m_V^2)} \tag{25}$$

which can be combined with the short distance contribution. The result improves agreement with the experimental mass difference,

$$\begin{aligned}\Delta m_\pi &= 4.0 - 4.4 \;\; MeV \;\; (TH) \\ &= 4.4 \;\; MeV \;\; (EXPT)\end{aligned} \tag{26}$$

In the chiral limit, the pion electromagnetic mass difference can also be computed using soft pion theorems. The mass difference is related to the vacuum value of the difference of vector and axial-vector current correlation functions. The Weinberg sum rules can then be applied to evaluate the mass difference. Our result is consistent with the classic results obtained from the sum rule approach [15].

Conclusions.

We have used the large N_C expansion to compute the B-parameter for the weak mixing of neutral B-mesons. The method requires a matching of the dual representations of the quark-gluon picture and the hadronic picture of the relevant amplitudes in leading order and next-to-leading order of the large N_C expansion. A consistent matching is obtained within the context of our approximations. However, the precise nature of the duality properties of the amplitudes involving heavy and light quark should be further clarified.

The $1/N_C$ prediction of the K-meson B-parameter, B_K, was reviewed and found to be in good agreement with the most recent values obtained using lattice field theory methods. The method can also be applied to the nonleptonic weak decay amplitudes of K-mesons. Good agreement is obtained for the delta I =3/2 amplitudes, and a significant part of the delta I =1/2 can be explained by a combination of long and short distance effects.

The $1/N_C$ expansion can also be applied to the computation of the electromagnetic mass difference of charged and neutral pions. The approximations can be systematically improved through the inclusion of vector and axial-vector meson contributions which saturate the short distance matching conditions. The results are in good agreement with experiment and with the original current algebra analysis of this mass difference.

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